Negative parity baryons in the $1/N_c$ expansion

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Overview of Large N_c QCD

QCD $\longrightarrow$ Non-abelian
Asymptotic freedom
Confinement

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$\Downarrow$

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- EFT
- Lattice QCD
- NJL models

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... Large N_c $\rightarrow$ expansion of QCD in terms of $1/N_c$

[1] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974)

How does QCD generalized to N_c colors look like? $(\mathrm{SU}(3) \rightarrow \mathrm{SU}(N_c))$

- Degrees of freedom increase

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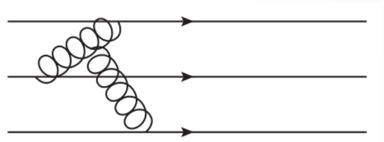

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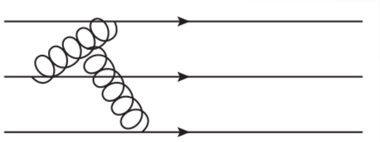
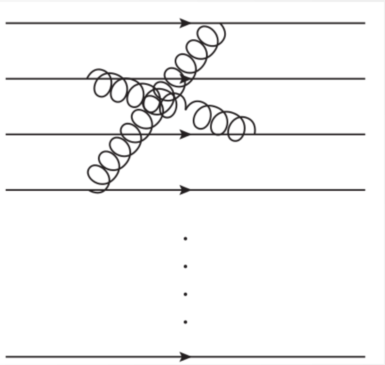


$\longrightarrow$



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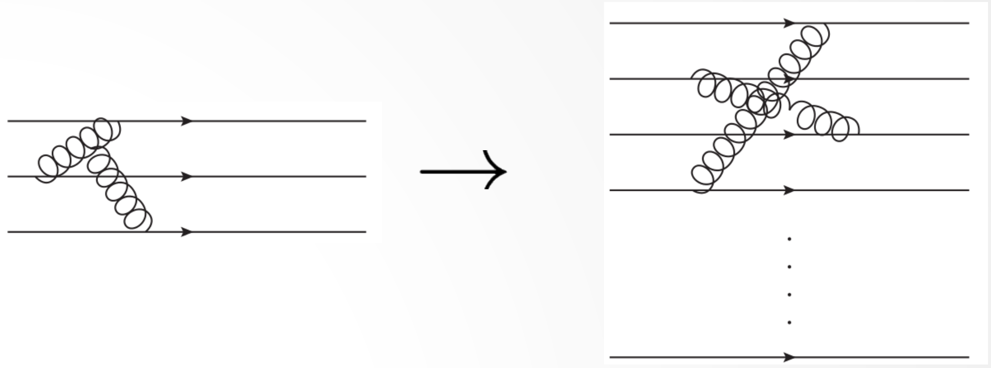

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$1/N_c$ is the only candidate for a perturbative expansion at **all** *energies*

Observables in Large N_c

The Large N_c expansion allows to organize operators by their effect on a given observable

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Large N_c was applied with great succes to ground state baryons (masses, magnetic moments, axial vector current, etc..) and mass relations.

L=1 baryons

States with same quantum numbers mix in terms of angles θ_J

Baryon	I	J^P	Mass (MeV)
$N_{3/2}$		$3/2^-$	1515 ± 5
$N_{1/2}$		$1/2^-$	1535 ± 10
$N'_{1/2}$	$1/2$	$1/2^-$	1658 ± 13
$N'_{3/2}$		$3/2^-$	1700 ± 50
$N_{5/2}$		$5/2^-$	1675 ± 5
$\Delta_{1/2}$	$3/2$	$1/2^-$	1675 ± 5
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Non-relativistic quark model

In the NR limit, the 2-body interactions contain three terms

$$H = H_0 + \sum_{i<j} V_{ij} = H_0 + V_{ss} + V_{so} + V_t \,.$$

spin-spin (V_{ss}), spin-orbit (V_{so}), quadrupole (V_t)

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$$v_\alpha = v_\alpha^0(r_{ij}) + v_\alpha^1(r_{ij})\tau_i^a\tau_j^a$$

$$V_{ss} = V_{ss}^0 + V_{ss}^1 = \sum_{i<j=1}^{N} v_{ss}(r_{ij})\vec{s}_i \cdot \vec{s}_j \,,$$

$$V_{so} = V_{so}^0 + V_{so}^1 = \sum_{i<j=1}^{N} v_{so}(r_{ij})\Big[(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j + 2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_i\Big] \,,$$

$$V_t = V_t^0 + V_t^1 = \sum_{i<j=1}^{N} v_t(r_{ij})\Big[3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j)\Big] \,.$$

Relevant operators to L=1 baryons

The mass matrix of a L=1 baryon can be written as a linear combination of the CCGL[2] operators

$$M = \sum_{i=1}^{18} c_i O_i = O_{\ell=0} + O_{\ell=1} + O_{\ell=2}$$

9 l.i. operators for $N_c = 3$

Complete set of time reversal even, rotationally invariant, isosinglet operators for non-strange excited baryons

organized by powers of $1/N_c$ in their matrix elements.

[2] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Rev. D **59**, 114008 (1999)

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$\Updownarrow$ We found that only 7 operators O_i are needed

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7 baryons masses
2 mixing angles
for L=1, non-strange

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Mass-angle relations

$$\frac{1}{2}(N_{1/2} - N'_{1/2})(3\cos 2\theta_1 + \sin 2\theta_1) + (N_{3/2} - N'_{3/2})\left(-\frac{3}{5}\cos 2\theta_3 + \sqrt{\frac{5}{2}}\sin 2\theta_3\right)$$
$$= -\frac{1}{2}(N_{1/2} + N'_{1/2}) + \frac{7}{5}(N_{3/2} + N'_{3/2}) - \frac{9}{5}N_{5/2} - 2\Delta_{1/2} + 2\Delta_{3/2}\,.$$

$$5(N_{1/2} - N'_{1/2})(\cos 2\theta_1 + 2\sin 2\theta_1) - 4(N_{3/2} - N'_{3/2})\left(2\cos 2\theta_3 + \sqrt{\frac{5}{2}}\sin 2\theta_3\right)$$
$$= 5(N_{1/2} + N'_{1/2}) - 8(N_{3/2} + N'_{3/2}) + 6N_{5/2}\,.$$

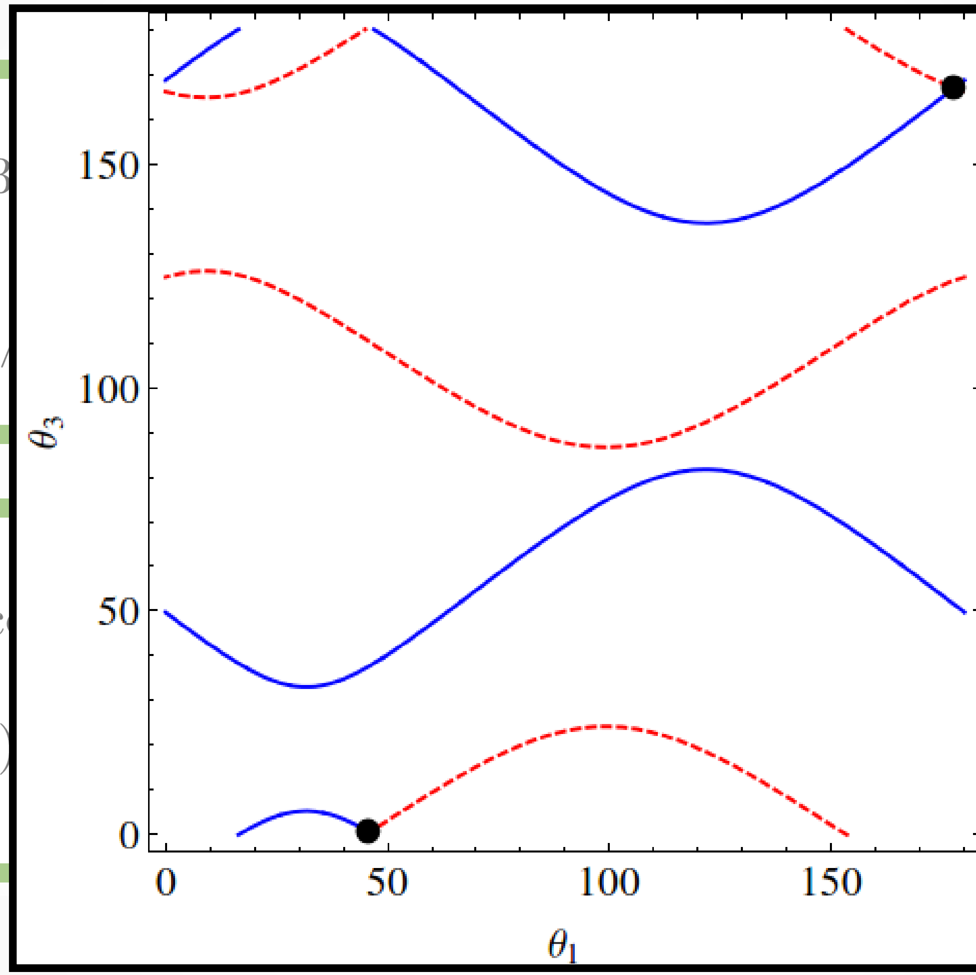
Mass-angle relations

$\frac{1}{2}(N_{1/2} - N'_{1/2})(3$ … $2\theta_3 + \sqrt{\frac{5}{2}}\sin 2\theta_3\Big)$

$= -\frac{1}{2}(N_{1/2} + N'_{1/}$ … $\Delta_{3/2}\,.$

$5(N_{1/2} - N'_{1/2})(\mathrm{c}$ … $\theta_3 + \sqrt{\frac{5}{2}}\sin 2\theta_3\Big)$

$= 5(N_{1/2} + N'_{1/2})$ …



Correlations in the (θ_1,θ_3) plane in the quark model with a general 2-body quark interaction.

Summary

- Large N_c is a tool that can be used to describe QCD phenomenology at non-perturbative energies

- Tool to compare models in a systematic way

In our work..

- We matched different versions of the NR quark model to the $1/N_c$ quark operator basis. And we obtained analytical and parameter free relations between the masses and angles of the L=1 baryons

Many thanks to JLab, the Jefferson Science Associates, HUGS and to the audience..

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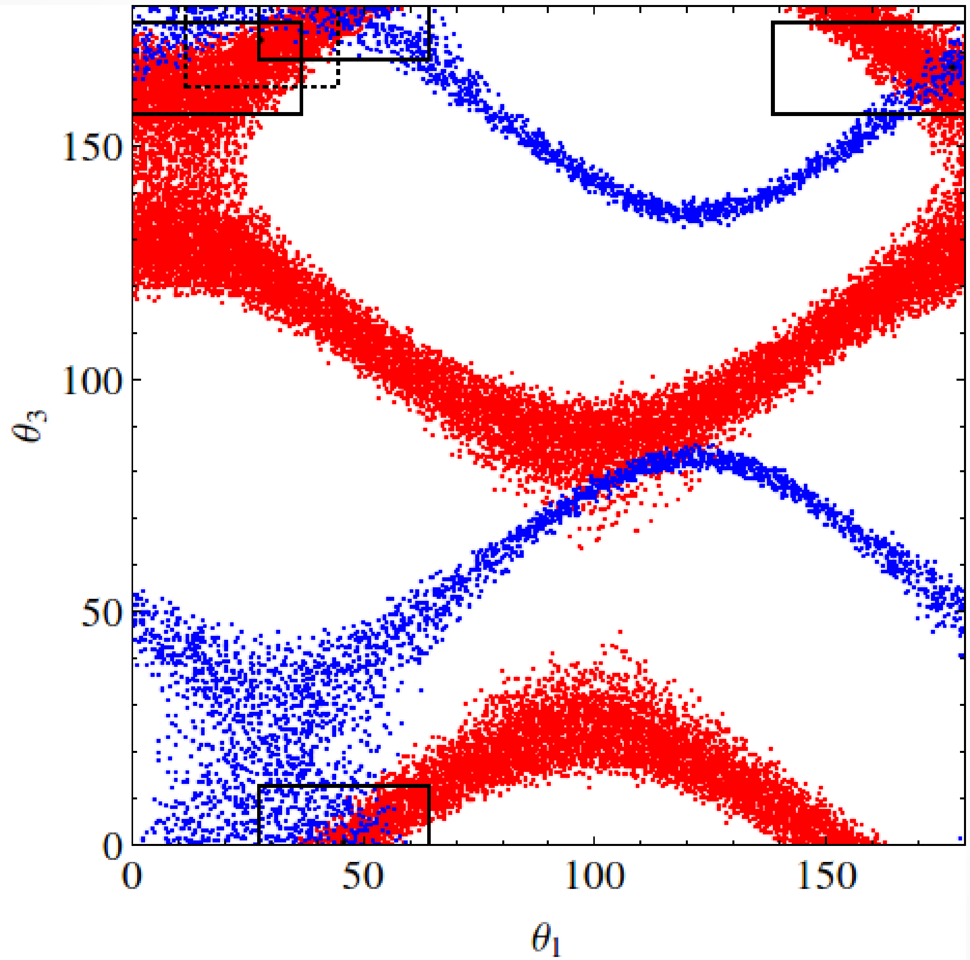
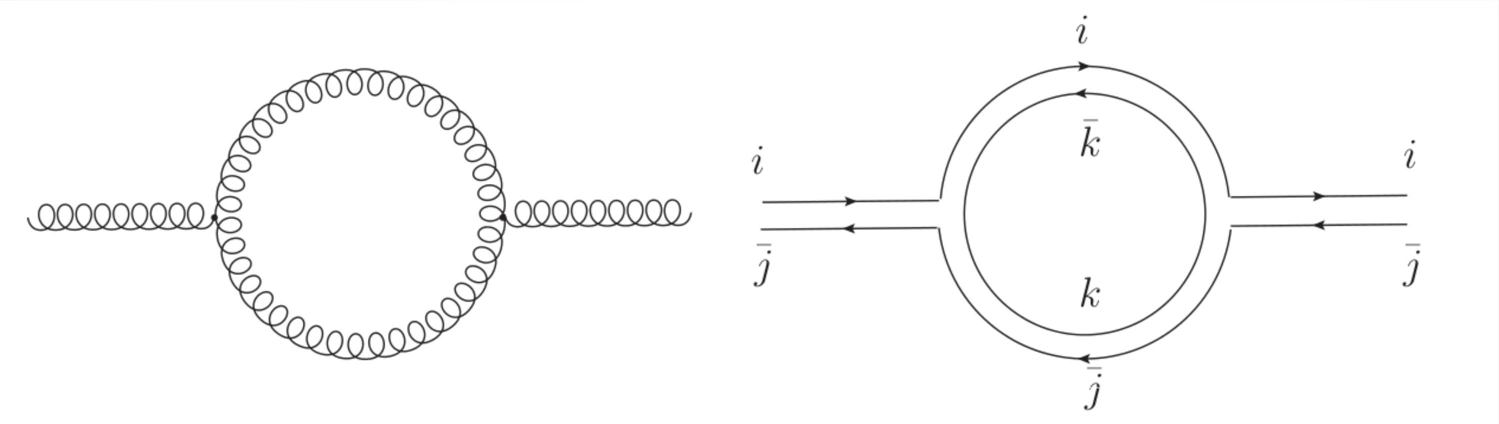
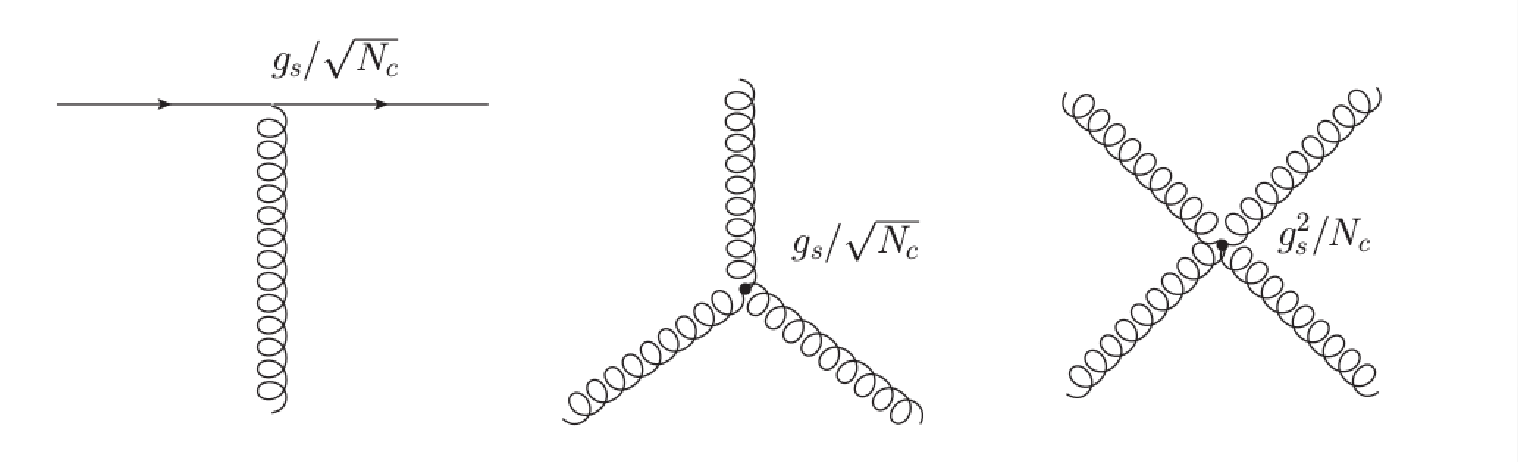


Scatter plot obtained including the experimental errors on the masses

Feynman diagrams in Large N_c





The quark model states are given by

$$|N_J; J_3 I_3\rangle = \frac{1}{2} \sum_{m,S_3} \left(\begin{array}{cc|c} 1 & \frac{1}{2} & J \\ m & S_3 & J_3 \end{array} \right) \left[\left(\xi^\rho_{S_3} \varphi^\rho_{I_3} - \xi^\lambda_{S_3} \varphi^\lambda_{I_3} \right) \Psi^\lambda_{1m} + \left(\xi^\rho_{S_3} \varphi^\lambda_{I_3} - \xi^\lambda_{S_3} \varphi^\rho_{I_3} \right) \Psi^\rho_{1m} \right], \quad (1)$$

$$|N'_J; J_3 I_3\rangle = \frac{1}{\sqrt{2}} \sum_{m,S_3} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & J \\ m & S_3 & J_3 \end{array} \right) \xi^S_{S_3} \left(\varphi^\rho_{I_3} \Psi^\rho_{1m} + \varphi^\lambda_{I_3} \Psi^\lambda_{1m} \right) , \quad (2)$$

$$|\Delta_J; J_3 I_3\rangle = \frac{1}{\sqrt{2}} \varphi^S_{I_3} \sum_{m,S_3} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & J \\ m & S_3 & J_3 \end{array} \right) \left(\xi^\rho_{S_3} \Psi^\rho_{1m} + \xi^\lambda_{S_3} \Psi^\lambda_{1m} \right) . \quad (3)$$